**Assignment 3**

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# Part One

## 1.1 Gram-Schmidt Orthogonalization

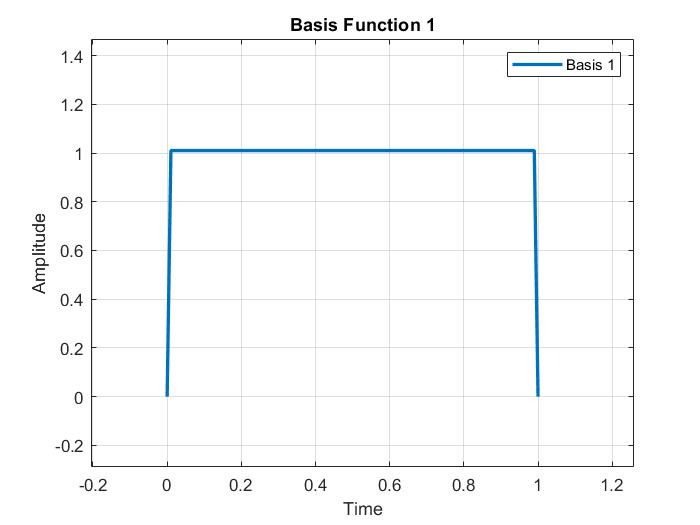


Figure Φ1 VS time after using the GM\_Bases function

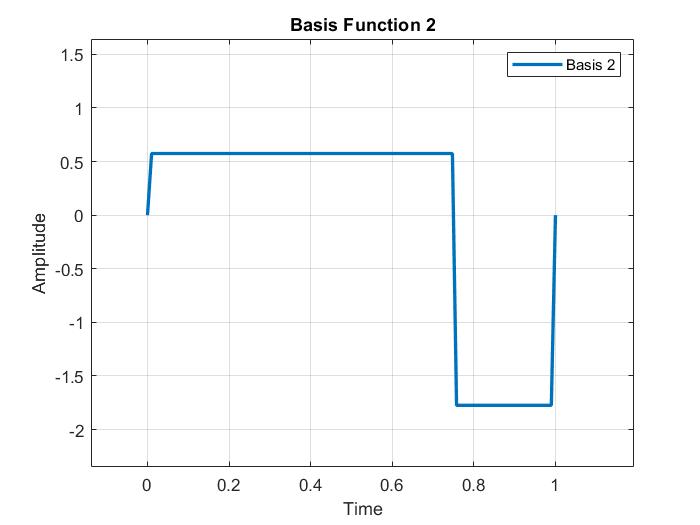


Figure Φ2 VS time after using the GM\_Bases function.

**Solution verification:**

So, the bases are orthogonal.

## 1.2 Signal Space Representation

Here we represent the signals using the base functions. “Without any noise”

A picture containing text, line, plot, screenshot

Description automatically generated

Figure Signal Space representation of signals s1, s2

## 1.3 Signal Space Representation with adding AWGN.

-the expected real points will be solid and the received will be hollow

**Case 1**:

|  |  |
| --- | --- |
|  | 50 sample  of signal#1  and signal#2  with noise |
|  | 100 sample  of signal#1  and signal#2  with noise |

Figure Signal Space representation of signals s1, s2 with E/σ¬2 =10dB

**Case 2**:

|  |  |
| --- | --- |
|  | 50 sample  of signal#1  and signal#2  with noise |
|  | 100 sample  of signal#1  and signal#2  with noise |

Figure Signal Space representation of signals s1,s2 with E/σ¬2 =0dB

**Case 3**:

|  |  |
| --- | --- |
|  | 50 sample  of signal#1  and signal#2  with noise |
|  | 100 sample  of signal#1  and signal#2  with noise |

Figure Signal Space representation of signals s1, s2 with E/σ¬2 =-5dB

## 1.4 Noise Effect on Signal Space

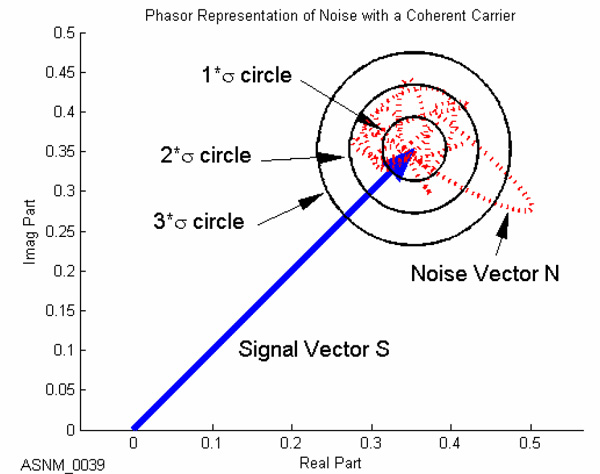
How does the noise affect the signal space?

Receiver will get

Then we find

The receiver will get a signal yij whose space vector near to the space vector of Sij “the space vector of the sent pulse.”, and we detect the actual signal space using gaussian probability.

Does the noise effect increase or decrease with increasing 𝜎2?

It’s noticed that with the increase of the variance of the noise, the signal to noise ratio decreases then the probability of error increases “the uncertainty increases” as observed from the following image.

# References

1. Matlab documentations for rectangularPulse function
2. [Wikipedia](https://en.wikipedia.org/wiki/Additive_white_Gaussian_noise#Effects_in_phasor_domain)

# Appendix A: Codes for Part One:

## A.1 Code for Gram-Schmidt Orthogonalization

function [phi1, phi2] = GM\_Bases(s1, s2)

    % Eenergy of s1 = dot(s1, s1)

    phi1 = s1 / sqrt(dot(s1, s1));

    % s21 = dot(s2, phi1) which is the projection of s2 onto phi1

    v2 = s2 - dot(s2, phi1) \* phi1;

    phi2 = v2 / sqrt(dot(v2, v2));

    % to normalize the bases

    phi1 = phi1 \* sqrt(length(s1));

    phi2 = phi2 \* sqrt(length(s2));

end

## A.2 Code for Signal Space representation

function [v1, v2] = signal\_space(s, phi1, phi2)

    v1 = dot(s, phi1) / length(s); %v1 = Si1 = dot<s, phi1> / ||s||

    v2 = dot(s, phi2) / length(s); %v2 = Si2 = dot<s, phi2> / ||s||

end

## A.3 Code for plotting the bases functions

% =================================================================

% Construct the signals

t = linspace(0, 1, 100);

s1 = rectangularPulse(0, 1, t);

s1(1) = 0; s1(end) = 0;

s2 = rectangularPulse(0, 0.75, t) - 1 \* rectangularPulse(0.75, 1, t);

s2(1) = 0; s2(end) = 0;

% =================================================================

% =================================================================

% REQUIREMENTS 1:

[phi1, phi2] = GM\_Bases(s1, s2);

% Plot the signals

figure('Name', 'Basis Functions', 'NumberTitle', 'off');

plot(t, phi1, 'LineWidth', 2);

legend('Basis 1');

xlabel('Time');

ylabel('Amplitude');

title('Basis Function 1');

grid on;

figure('Name', 'Basis Functions', 'NumberTitle', 'off');

plot(t, phi2, 'LineWidth', 2);

legend('Basis 2');

xlabel('Time');

ylabel('Amplitude');

title('Basis Function 2');

grid on;

## A.4 Code for plotting the Signal space Representations

[s1\_v1, s1\_v2] = signal\_space(s1, phi1, phi2);

[s2\_v1, s2\_v2] = signal\_space(s2, phi1, phi2);

% Plot the signal

figure('Name', 'Signal Space Representation', 'NumberTitle', 'off');

plot([0 s1\_v1], [0 s1\_v2], '-o', 'MarkerIndices', [2 2], 'LineWidth', 2);

hold on;

plot([0 s2\_v1], [0 s2\_v2], '-o', 'MarkerIndices', [2 2], 'LineWidth', 2);

legend('Signal 1', 'Signal 2');

xlabel('Phi1');

ylabel('Phi2');

title('Signal Space Representation');

grid on;

## A.5 Code for effect of noise on the Signal space Representations

% signal\_space\_with\_noise.m

function r = signal\_space\_with\_noise(s, sigma)

    % Generate AWGN

    noise = sigma \* randn(1, length(s));

    % Add noise to the signals

    r = s + noise;

end

% ==========================================================

% plot\_signal\_with\_noise.m

function plot\_signal\_with\_noise(testCase, s1\_v1, s1\_v2, s2\_v1, s2\_v2, s1, s2, phi1, phi2)

    % Draw the signal space representation of the signals before adding noise

    figure('Name', 'Signal Points with Noise', 'NumberTitle', 'off');

    scatter(s1\_v1, s1\_v2, 100, 'r', 'filled');

    hold on;

    scatter(s2\_v1, s2\_v2, 100, 'b', 'filled');

    % E / sigma^2 list in dB

    EoSigma = [-5, 0, 10];

    Es1 = dot(s1, s1) / length(s1);

    Es2 = dot(s2, s2) / length(s2);

    disp(Es1);

    disp(Es2);

    sigma1 = sqrt(Es1 ./ db2mag(EoSigma));

    sigma2 = sqrt(Es2 ./ db2mag(EoSigma));

    for i = 1 : 100

        r1 = signal\_space\_with\_noise(s1, sigma1(testCase));

        r2 = signal\_space\_with\_noise(s2, sigma2(testCase));

        % Calculate signal space representation of the generated samples

        [r1\_v1, r1\_v2] = signal\_space(r1, phi1, phi2);

        [r2\_v1, r2\_v2] = signal\_space(r2, phi1, phi2);

        % Draw the signal space representation of the signals after adding noise

        scatter(r1\_v1, r1\_v2, [], [0.6350 0.0780 0.1840]);

        scatter(r2\_v1, r2\_v2, [], [0.3010 0.7450 0.9330]);

    end

    legend("Signal 1", "Signal 2", "Signal 1 with Noise", "Signal 2 with Noise");

    xlabel('Phi1');

    ylabel('Phi2');

    title('Signal Points with Noise with E/sigma^2 = ' + string(EoSigma(testCase)) + 'dB');

    grid on;

end

% =================================================================

% Function calls to plot the signals with noise in main.m

plot\_signal\_with\_noise(1, s1\_v1, s1\_v2, s2\_v1, s2\_v2, s1, s2, phi1, phi2);

plot\_signal\_with\_noise(2, s1\_v1, s1\_v2, s2\_v1, s2\_v2, s1, s2, phi1, phi2);

plot\_signal\_with\_noise(3, s1\_v1, s1\_v2, s2\_v1, s2\_v2, s1, s2, phi1, phi2);

% =================================================================